

Cl Haf 2011

I) a) A(3, 11) B(9, -1)

$$m_{AB} = \frac{11 - -1}{3 - 9} = \frac{12}{-6} = -2$$

b) os L $m_1 \cdot m_2 = -1$ $\Rightarrow m_2 = \frac{1}{2}$

mindig passzáljon $B(9, -1)$

$$\begin{aligned}y + 1 &= \frac{1}{2}(x - 9) \\2y + 2 &= x - 9 \\2y - x + 11 &= 0\end{aligned}$$

c) i) $L_1: 2y - x + 11 = 0$
 $x = 2y + 11$

$$\begin{aligned}L_2: 6x + 7y + 10 &= 0 \\6(2y + 11) + 7y + 10 &= 0 \\12y + 66 + 7y + 10 &= 0 \\19y &= -76 \\y &= \frac{-76}{19} = -4\end{aligned}$$

$$\begin{aligned}x &= 2y + 11 \\&= -8 + 11 \\&= 3\end{aligned}$$

C(3, -4)

ii) B(9, -1) C(3, -4)

$$\sqrt{(9-3)^2 + (-1-(-4))^2} \\ \sqrt{36 + 9} = \sqrt{45}$$

III) Carrelsgant BC

$$x_1 = \frac{9+3}{2} \\ = 6$$

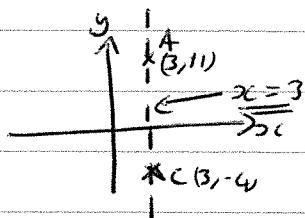
$$y_1 = \frac{-1 + -4}{2} \\ = -2.5$$

$$\text{Carrelsgant} = (6, -2.5)$$

IV) $A(3,11) \quad C(3,-4)$

Halbsäule AC: $x=3$

grapher
diagram



2 a) $\frac{9}{\sqrt{3}-1} + \frac{7}{\sqrt{3}+1}$

$$\frac{9(\sqrt{3}+1) + 7(\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{9\sqrt{3} + 9 + 7\sqrt{3} - 7}{3 - 1}$$

$$= \frac{16\sqrt{3} + 2}{2} = 8\sqrt{3} + 1$$

b) $\frac{90}{\sqrt{3}} - \sqrt{6} \times \sqrt{8} - (2\sqrt{3})^3$

$$\frac{90\sqrt{3}}{3} - 4\sqrt{3} - 24\sqrt{3}$$

$$30\sqrt{3} - 4\sqrt{3} - 24\sqrt{3} = 2\sqrt{3}$$

3 $y = 3x^2 - 9x + 1$

$P \rightarrow x=2$

$$\frac{dy}{dx} = 6x - 9$$

$$y = 3(2)^2 - 9(2) + 1 = -5$$

$$P(2, -5)$$

gradient par fü $x=2$

$$6(2) - 9 = \underline{\underline{3}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3(x - 2)$$

$$y + 5 = 3x - 6$$

$$y = 3x - 11$$

$$4 \quad -x^2 + 6x - 7$$

$$-(x^2 - 6x) - 7$$

$$-(x-3)^2 - 7 + 9$$

$$-(x-3)^2 + 2$$

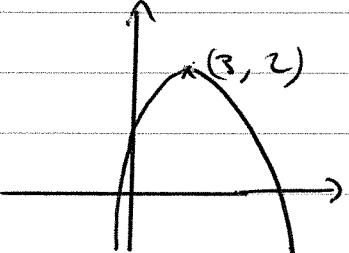
$$-(x-3)^2 = -(x-3)(x-3)$$

$$= -(x^2 - 6x + 9)$$

gwerth mygf 2 pan fo $x = 3$

$$= -x^2 + 6x - 9$$

o o point mygf $(3, 2)$



$$5 \quad y = x^2 + (4k+3)x + 7$$

$$y = x + k$$

$$a) \quad x + k = x^2 + (4k+3)x + 7$$

$$x^2 + (4k+2)x + (7-k) = 0$$

os 2 bigt gwahanady

$$b^2 - 4ac > 0$$

$$a = 1$$

$$(4k+2)^2 - 4(1)(7-k) > 0$$

$$b = 4k+2$$

$$16k^2 + 16k + 4 - 28 + 4k > 0$$

$$c = 7-k$$

$$16k^2 + 20k - 24 > 0 \quad (\div 4)$$

$$4k^2 + 5k - 6 > 0$$

$$b) \quad 4k^2 + 5k - 6 > 0$$

$$4k^2 + 8k - 3k - 6 > 0$$

$$4k(k+2) - 3(k+2) > 0$$

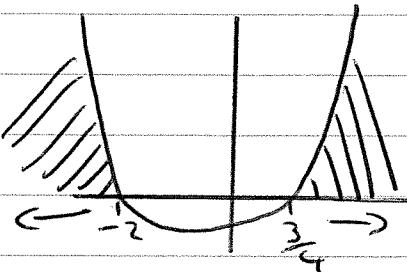
$$\begin{array}{r} +1x \\ +5 \\ \hline -24 \end{array}$$

$$(8, -3)$$

$$(4K - 3)(K + 2) > 0$$

$$4K - 3 = 0 \text{ then } K + 2 = 0$$

$$K = \frac{3}{4} \text{ then } K = -2$$



$$\therefore K > \frac{3}{4} \text{ or } K < -2$$

6 a) $y = 7x^2 - 5x + 2$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{7(x+h)^2 - 5(x+h) + 2 - (7x^2 - 5x + 2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{7x^2 + 14xh + 7h^2 - 5x - 5h + 2 - 7x^2 + 5x - 2}{h} \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{14xh + 7h^2 - 5h}{h}$$

$$= \lim_{h \rightarrow 0} 14x + 7h - 5$$

$$\text{Put } h \rightarrow 0 \quad \frac{dy}{dx} = 14x - 5$$

b) $y = 4x^{\frac{2}{3}} - \frac{9}{x} - 6$

$$y = 4x^{\frac{2}{3}} - 9x^{-1} - 6$$

$$\frac{dy}{dx} = 4\left(\frac{2}{3}\right)x^{-\frac{1}{3}} + 9x^{-2}$$

$$= \frac{8}{3}x^{-\frac{1}{3}} + \frac{9}{x^2}$$

7 a) $(3 + 2x)^4 = 3^4 + 4(3^3)(2x) + 6(3)^2(2x)^2 + 4(3)(2x)^3 + (2x)^4$

$$= 81 + 216x + 216x^2 + 96x^3 + 16x^4$$

$$b) \left(1 + \frac{x}{4}\right)^n = 1^n + n(1)^{n-1} \left(\frac{x}{4}\right) + \frac{n(n-1)(1)^{n-2} \left(\frac{x}{4}\right)^2}{2!} + \dots$$

$$= 1 + \frac{n x}{4} + \frac{n(n-1)}{32} x^2$$

Cyfarnod $x^2 \rightarrow \frac{n(n-1)}{32}$

Cyfarnod $x \rightarrow \frac{n}{4}$

∴ $S\left(\frac{n}{4}\right) = \frac{n(n-1)}{32}$

$$32 \times \frac{S_n}{4} = n^2 - n$$

$$40n = n^2 - n$$

$$0 = n^2 - 41n$$

$$0 = n(n-41)$$

$$n = 0 \times \text{nen} \quad \underline{\underline{n = 41}}$$

8 a) $f(x) = p x^3 - x^2 - 31x + q$

$$f(-2) = 0$$

$$f(1) = -36$$

∴ $p(-2)^3 - (-2)^2 - 31(-2) + q = 0$

$$-8p - 4 + 62 + q = 0$$

$$-8p + 58 + q = 0 \quad q = 8p - 58 \quad ①$$

$$p(1)^3 - (1)^2 - 31(1) + q = 0$$

$$p - 1 - 31 + q =$$

$$p - 32 + q = \quad q = \quad ②$$

$$\textcircled{1} = \textcircled{2}$$

$$\begin{array}{rcl} 8p - 58 & = & -p - 4 \\ 9p & = & 54 \\ p & = & \underline{\underline{6}} \end{array}$$

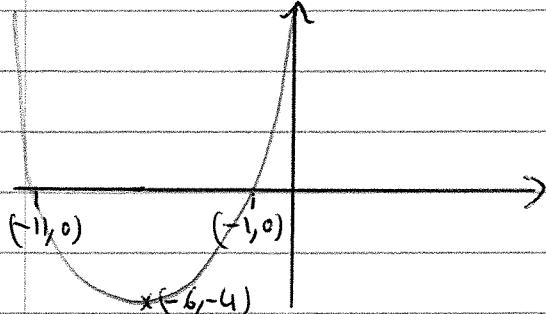
$$\begin{array}{l} q = 8p - 58 \\ = 48 - 58 \\ q = \underline{\underline{-10}} \end{array}$$

b) If factors $6x^3 - x^2 - 31x - 10$

$$f(-2) = 0 \quad (x+2) \text{ is a factor}$$

$$\begin{aligned} (x+2)(6x^2 - 13x - 5) &= 0 \\ (x+2)(6x^2 - 15x + 2x - 5) &= 0 \quad -13 + x - 30 \\ (x+2)(3x(2x-5) + 1(2x-5)) &= 0 \quad -15 + 2 \\ (x+2)(3x+1)(2x-5) &= 0 \\ (x = -2 \text{ new } x = -\frac{1}{3} \text{ new } x = \frac{5}{2}) &\leftarrow \text{diagram} \\ &\text{(and median given)} \end{aligned}$$

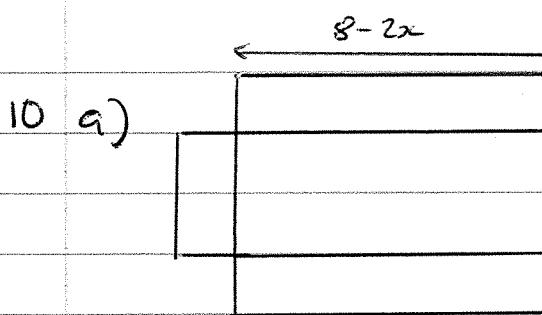
9 a) $y = R(x+3)$



b) $y = rR(x)$

for $r = -\frac{1}{2}$

$$\therefore y = -\frac{1}{2}R(x)$$



Cylant

$$\begin{aligned}
 V &= x \times (5-2x) \times (8-2x) \\
 &= x(40 + 4x^2 - 16x - 10x) \\
 &= x(4x^2 - 26x + 40) \\
 &= 4x^3 - 26x^2 + 40x
 \end{aligned}$$

b) $\frac{dV}{dx} = 12x^2 - 52x + 40$

gwerth maks/min
pan lu $\frac{dV}{dx} = 0$

$\therefore 12x^2 - 52x + 40 = 0 \quad (\div 4)$

$$\begin{array}{r}
 3x^2 - 13x + 10 = 0 \\
 3x^2 - 10x \quad -3x + 10 = 0 \\
 \hline
 \quad -13 \quad | 30
 \end{array}$$

$$x(3x-10) - 1(3x-10) = 0$$

$$(x-1)(3x-10) = 0$$

$x = 1$ neu $x = \frac{10}{3}$, gan bed $x < 2.5$

$$\frac{d^2V}{dx^2} = 24x - 52$$

pan lu $x = 1$

$$\frac{d^2V}{dx^2} = 24 - 52 < 0$$

\therefore maks

Gwerth maks am $V = 4(1)^3 - 26(1)^2 + 40(1)$

$$\begin{aligned}
 &= 4 - 26 + 40 \\
 &= \underline{\underline{18}}
 \end{aligned}$$