Question 1 C2 - Half 2013

$h = \frac{2-0}{4} = \frac{2}{4}$

$= \underline{0.5}$

x	$\frac{1}{2+x^3}$	
0	0.5	$1/2$
0.5	0.4705882353	$8/17$
1	$0.\dot{3}$	$1/3$
1.5	0.1860465116	$8/43$
2	0.1	$1/10$

B2

M1

$$I \approx \frac{0.5}{2}\left\{0.5 + 2\left(0.4705882353 + 0.3333333 + 0.1860465116\right) + 0.1\right\}$$

$I \approx 0.644984038$

$I \approx 0.645$ (3 dp degol) A1 cao

Question 2

2 a (i) $6\cos\theta + 5\tan\theta = 0$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\therefore \quad 6\cos\theta + 5\left(\dfrac{\sin\theta}{\cos\theta}\right) = 0$ M1

$\times \cos\theta$ $\quad 6\cos^2\theta + 5\sin\theta = 0$

$\cos^2\theta = 1 - \sin^2\theta.$

M1

$\therefore \quad 6(1 - \sin^2\theta) + 5\sin\theta = 0$

$6 - 6\sin^2\theta + 5\sin\theta = 0$

$0 = 6\sin^2\theta - 5\sin\theta - 6$ A1

$x = \sin\theta$

$6x^2 - 5x - 6 = 0$

$(3x + 2)(2x - 3) = 0$

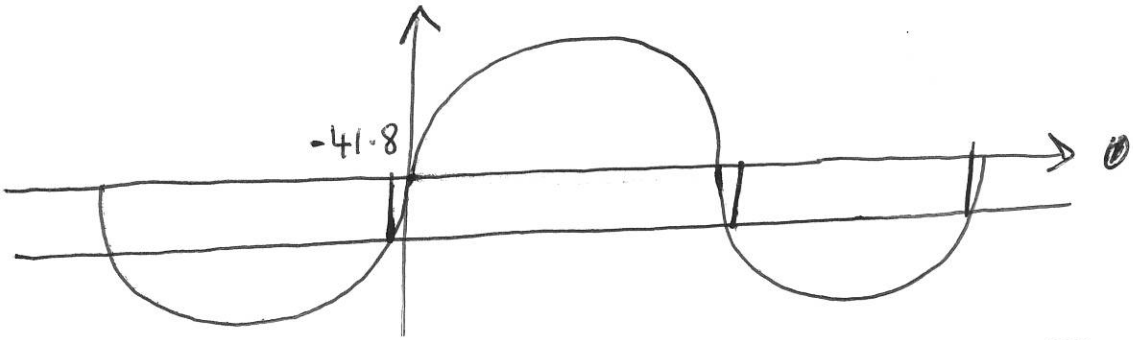
M1

$x = -\dfrac{2}{3}$ <u>neu</u> $x = \dfrac{3}{2}.$

$\therefore \quad \sin\theta = -\dfrac{2}{3}$ neu $\sin\theta = \dfrac{3}{2}$ (anhosibl)

A1

$\therefore \quad \theta = \sin^{-1}\left(\frac{-2}{3}\right) = -41.8^\circ$



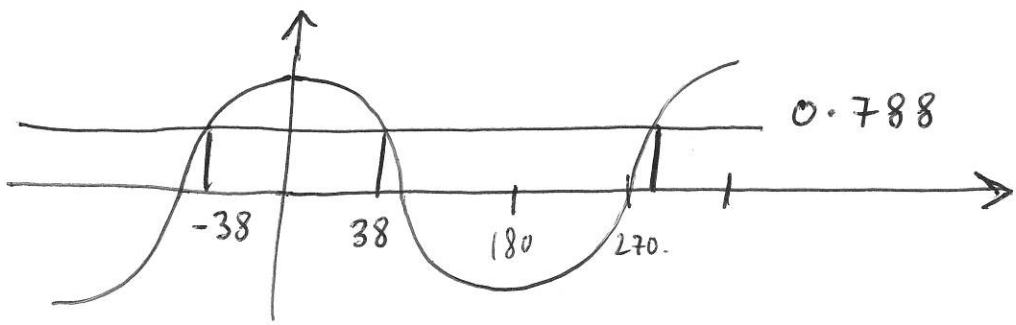
$\theta = 180^\circ + 41.8 = \underline{221.8^\circ}$ (B1)

$360^\circ - 41.8 = \underline{318.2^\circ}$ (B1)

(2) (b) $\cos(2x - 60^\circ) = 0.788$

$2x - 60^\circ = \cos^{-1}(0.788)$

$2x - 60^\circ = -38^\circ, 38^\circ, 322^\circ$ (B1)



$2x = 22^\circ, 98^\circ, 382$ (B1)

$\underline{x = 11^\circ, 49^\circ,} \cancel{191^\circ}$ (B1)

Question 3

(a) Rhool Cosine $a^2 = b^2 + c^2 - 2bc \cos A$

$$(x+2)^2 = x^2 + (x-2)^2 - 2x(x-2)\cos B\hat{A}C$$

$$\cancel{x^2} + 4x + \cancel{4} = \cancel{x^2} + x^2 - 4x + \cancel{4} - (2x^2 - 4x)\cos A$$

$$-x^2 + 8x = -(2x^2 - 4x)\cos A$$

$$\frac{x^2 - 8x}{2x^2 - 4x} = \cos A$$

$\div x$

$$\frac{x - 8}{2x - 4} = \cos A.$$

(b)

$$\frac{x-8}{2x-4} = \cos 120^\circ$$

M1

$$\frac{x-8}{2x-4} = -\frac{1}{2}.$$

$$x - 8 = -\frac{1}{2}(2x-4)$$

$$x - 8 = -x + 2$$

$$2x = 10$$

$$x = 5$$

A1

③ (b) (ii) $\frac{\sin B}{x-2} = \frac{\sin A}{x+2}$

$$\frac{\sin B}{3} = \frac{\sin 120}{7}$$ (M1)

$$\sin B = \frac{3 \times \sin 120}{7}$$

$$\sin B = \frac{3\sqrt{3}}{14} = 0.3711537$$

$$B = \sin^{-1}\left(\frac{3\sqrt{3}}{14}\right) = \underline{21.8^\circ}$$ (A1)

④ (a) $S_n = a + (a+d) + \ldots\ldots + (a+(n-1)d)$ ① (B1)

$S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots\ldots + a$ ②

① + ② $2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots\ldots + [2a+(n-1)d]$ (M1)

$$2S_n = n[2a+(n-1)d]$$

$$S_n = \frac{n}{2}[2a+(n-1)d]$$ (A1)

(4)(b) $\frac{n}{2}\left(2a + (n-1)d\right) = S_n$

$S_{10} = \frac{10}{2}\left(2a + 9d\right) = 115$

$5\left(2a + 9d\right) = 115$

$10a + 45d = 115$ (1) (B1)

$S_{14} = 115 + 130 = 245$

$\frac{14}{2}\left(2a + 13d\right) = 245$

$14a + 91d = 245$ (2) (M1)

(A1)

$2a + 9d = 23$ (1)

$2a + 13d = 35$ (2)

(2) − (1) $4d = 12$ (M1)

$\underline{d = 3}$

In (1), $2a + 9 \times 3 = 23$

$2a + 27 = 23$

$2a = -4$ (A1)

$\underline{a = -2}$

Question 5

$n = 18$, $a = 100$.

(a) $S_n = \dfrac{a(1-r^n)}{1-r}$ $\qquad r = \dfrac{n+1}{n} = \dfrac{80}{100} = 0.8$ (B1)

$= \dfrac{100(1-0.8^{18})}{1-0.8} = 490.992$ (M1)

$= 491$ (A1)

(b) (i) $a + ar + ar^2 + \ldots\ldots + ar^{n+1}$

$ar = -20$ (B1) $\qquad S_\infty = \dfrac{a}{1-r}$

$\dfrac{a}{1-r} = 64$ (B1)

$a = 64 - 64r$

$a = \dfrac{-20}{r}$

$\dfrac{-20}{r} = 64 - 64r$ $(\times r)$

$-20 = 64r - 64r^2$ $\boxed{M1}$

$64r^2 - 64r - 20 = 0$ $(\div 4)$

$16r^2 - 16r - 5 = 0$ $\boxed{A1}$

(ii) $(4r - 5)(4r + 1)$

$r = 5/4$, $r = -1/4$ (B1)

$\therefore r = -1/4$, as $|r| < 1$. (E1)

Cwestiwn 6

(a) $\int x^{1/4} + 2x^{-5} \, dx$

$$= \frac{x^{5/4}}{5/4} + \frac{2x^{-4}}{-4} + C$$

$$= \frac{4x^{5/4}}{5} - \frac{x^{-4}}{2} + C$$

(b) (i) gyfesurynnau A a B pan mae $y = x^2 + 3$ yn croesi $y = 4x$

$x^2 + 3 = 4x$ (M1)

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$ (m1)

$\underline{x = 3}$ $\underline{x = 1}$ (A1)

(ii) Triongl bach. $x = 1$, $y = 4 \times 1 = 4$.

A $(1, 4)$ $\frac{\text{Sail} \times \text{Uchder}}{2} = \frac{1 \times 4}{2} = \underline{2 \text{ uned}^2}$ (B1)

Rhwng A a B.

$$\int_1^3 x^2 + 3 \, dx = \left[\frac{x^3}{3} + 3x\right]_1^3 = 18 - 3\tfrac{1}{3}$$

$$= \underline{14\tfrac{2}{3} \text{ uned}^2}$$

Cyfanswm: $14\tfrac{2}{3} + 2 = 16\tfrac{2}{3} = \underline{16.\dot{6} \text{ uned}^2}$

Question 7

(a.) $p = \log_a x \;, \longrightarrow \; x = a^p$

$q = \log_a y. \longrightarrow \; y = a^q$ (B1)

$xy = a^p \times a^q$

$xy = a^{p+q}$ (B1)

$\log_a xy = p + q$ (B1)

$\log_a xy = \log_a x + \log_a y$

(b.) $5^{2-3x} = 8.$

$\log 5^{2-3x} = \log 8.$

$(2-3x)\log 5 = \log 8.$

$2 - 3x = \dfrac{\log 8}{\log 5.}$ (M1)

$-3x = \dfrac{\log 8}{\log 5} - 2.$

(A1)

$-3x = 1.292029674. - 2.$

$-3x = -\cancel{0.430676558} - \cancel{0.646014}.....$

$x = \cancel{-0.431}$ (3 dec degree)

0.236 (A1)

(7c) $\log 90x^2 - \log\left(\frac{5}{x}\right) = \frac{1}{2}\log 144x^8$

$$\log\left(\frac{90x^2}{\frac{5}{x}}\right) = \log 12x^4.$$

$$\log\left(\frac{90x^3}{5}\right) = \log 12x^4.$$

$$\log 18x^3 = \log 12x^4.$$

$$18\cancel{x^3} = 12x^{\cancel{4}}.$$

$$18 = 12x$$

$$\frac{18}{12} = x$$

$$\underline{\underline{1.5 = x}}$$

(8) $x^2 + y^2 + 2x - 6y - 15 = 0$

$$(x^2 + 2x) + (y^2 - 6y) - 15 = 0$$

$$(x+1)^2 - 1 + (y-3)^2 - 9 - 15 = 0$$ (M1)

$$(x+1)^2 + (y-3)^2 - 25 = 0.$$

Centre $(-1, 3)$ (B1) radius $= 5.$ (A1)

(8) (b) (i) $y = -x + 9$

canol C_1 yw $(-1, 3)$. Amnewid i L_1

$3 \neq -1 + 9$

$3 \neq 8$

felly nid yw L_1 yn mynd trwy

$y = -x + 9$ felly nid yw yn diameter. (B1)

(ii)

$x^2 + y^2 + 2x - 6y - 15 = 0$

$x^2 + (-x+9) + 2x - 6(-x+9) - 15 = 0$. (M1)

$\cancel{x^2} + \cancel{x^2} - \cancel{18x} + 81 + \cancel{2x} + \cancel{6x} - 54 - 15 = 0$

$2x^2 - 10x + 12 = 0$

$x^2 - 5x + 6 = 0$. (A1)

$(x-3)(x-2) = 0$

$x = 3, \quad x = 2$. (A1)

$y = -3 + 9 \qquad y = -2 + 9$

$y = 6 \qquad y = 7$. (A1)

$\underline{\underline{(3, 6)}} \qquad \underline{\underline{(2, 7)}}$

(c) pellter rhwng A a B pellter $= \sqrt{(11 - -1)^2 + (8 - 3)}$

Cyfanswm 2 radius $5 + 6 = 11$ (M1) $= \sqrt{12^2 + 5^2}$

Pellter lleiaf rhwng C_1 a C_2 yw. $= \sqrt{144 + 25}$

$= \sqrt{169} = \underline{\underline{13}}$ (B1)

$13 - 11 = 2$ (A1)

Question 9

(a) Amoyneked Triangl $= \frac{1}{2} \times ab \times \sin C$. (M1)

$= \frac{1}{2} \times 7.2 \times 7.2 \times \sin 1.1 = \underline{23.1 \text{ cm}^2}$ (A1)

b)

Sector $= \frac{1}{2} ab\,\phi$

$19.44 = 0.5 \times 7.2^2 \times \phi$ (M1)

$\phi = \underline{0.75}$ (A1)

Hyd Arc $= r\phi = 7.2 \times 0.75 = \underline{5.4 \text{ cm}}$ (A1)